Solutions and Grading Key, Creative Response

Problem 1

(a, 8)

The force F_o must accelerate N blocks each of mass m:
$F_o = Nma$, so that $a = F_o/Nm$. (4)

Let T be the tension dragging N-n blocks to the right. $F = ma$ gives
$T = (N-n)ma$ (2)

Using a,
$T = (N-n)\, F_o/N = F_o(1 - n/N)$. (2)

(b, 12)

Each block has exerted on it the friction force μmg. (2)

$F = ma$ applied the the entire N-block system gives
$F_o - N\mu mg = Nma$ (2)

Solving for acceleration,
$a = (F_o/Nm) - \mu g$ (2)

Let T be the tension dragging N-n blocks to the right. $F=ma$ gives
$T - (N-n)\mu mg = (N-n)ma$ (2)

so that upon substituting for a,
$T = (N-n)m\,[\mu g + (F_o/Nm) - \mu g\,]$ (2)

which gives the final answer,
$T = F_o(1 - n/N)$, the same as without friction. (2)

Problem 2

(a, 10) Work energy theorem: $W = \Delta K$ (2)

Setting up the work energy theorem,

$[\, mg \sin\theta - \mu N \,] L = \frac{1}{2} m v^2 - 0$, (4)

where N is the normal force. But from $F = ma$ normal to the roof, $N = \mu mg \cos\theta$ (2)

thus, $L = \frac{1}{2} v^2 / [g (\sin\theta - \mu \cos\theta)]$
and numerically,

$L = \frac{1}{2}(3.5 \text{ m/s})^2 / [(9.8 \text{ m/s}^2)(\sin 30° - \frac{1}{4} \cos 30°)] = 2.2$ m. (2)

(-1 for incorrect units; -1 for gross neglect of significant figures)

Alternative method:

$F = ma$ tangent to the roof gives $a = g \sin\theta - \mu N$ [4]

$= g \sin\theta - \mu g \cos\theta$ [2]

$= 2.8 \text{ m/s}^2$. [1]

Using $v^2 - v_o^2 = 2a(x - x_o)$ [2]

leads to $L = v^2 / 2a = 2.2$ m. [1]

(b, 10) Projectile problem:

$v_{oy} = (3.5 \text{ m/s}) \sin 30° = 1.75$ m/s, (1)

$v_{ox} = (3.5 \text{ m/s}) \cos 30° = 3.03 \text{ m/s} \approx 3.0$ m/s. (1)

For the x-direction,

$x = v_{ox}\, t$, (2)

For the y-direction (taking down positive)

$y = v_{oy}\, t + \frac{1}{2} g t^2$. (2)

Solve the equations simultaneously by some appropriate strategy. For instance, eliminating t in the y-equation by using the x-equation: $y = v_{oy} (x/v_{ox}) + \frac{1}{2} g (x/v_{ox})^2$ (2)

Putting in numerical values, solving the quadratic equation correctly, choosing the + root in the quadratic formula (with the − root one gets an $x < 0$, which in *inside* the house), one finds $x = 1.9$ m. (2)

Yes, the box lands in the flowers.

Problem 3

(a, 3)
By conservation of energy,
$\frac{1}{2}I\omega_o^2 + mgh = K_I + mgR$
so that
$K_I = \frac{1}{2}I\omega_o^2 + mg(h - R)$ (3)

(b, 17)

Let (v_x, v_y) = velocity of the ball immediately *after* the collision is over.
Given:
$\beta K_I = \frac{1}{2}m(v_x^2 + v_y^2)$ (2)
Since β and K_I are known, if either of v_x or v_y is known then so is the other. Let us find v_x.
(Note that linear momentum is *not* conserved.)

In the collision, the angular frequency goes from ω_o to 0.
Let f be the average force of friction that acts horizontally on the ball during the collision. Then from torque = $\Delta L/\Delta t$,
$fR = I\omega_o/\Delta t$
where Δt is the collision time. (5)

Now apply $F = m\,\Delta v/\Delta t$ in the x-direction.
The x-component of velocity changes from 0 to v_x during time Δt, and the horizontal force is f:
$f = m\,v_x/\Delta t$. (5)

From the torque equation we have $f\Delta t = I\omega_o/R$, so combining the torque and force equations, (2)
$I\omega_o/R = m\,v_x$
and solving for v_x,
$v_x = I\omega_o/mR$ (2)

Now v_x is known. Thus, so is v_y,
$v_y = [\,2\beta K_I/m - v_x^2\,]^{1/2}$ (1)

Problem 4

(a, 7) From $F = ma$, since the orbit is circular so that the acceleration is v^2 / R,

$(1 + \Gamma)(GMm/R^2) = mv^2 / R$ (2)

Using $v = 2\pi R/T$ where T is the period, (2)

$F = ma$ becomes (after some re-arrangement)

$T^2 = (4\pi^2 / GM)R^3 (1 + \Gamma)^{-1}$

Notice that $T_o^2 = (4\pi^2 / GM)R^3$, where T_o is the period of the planet's orbit with purely the Newtonian force. Hence, $T = T_o(1 + \Gamma)^{-1/2}$ (2)

Since $\Gamma << 1$, by the binomial expansion

$T \approx T_o(1 - \frac{1}{2}\Gamma)$ (1)

(b, 7)

In elapsed time

$\Delta t = T_o$, the planet turns through the angle $2\pi + \delta$ (2)

This angle can also be equated to ωT_o where ω is the planet's orbital angular velocity, viz.,

$\omega = 2\pi / T$

$= (2\pi / T_o)(1 - \frac{1}{2}\Gamma)^{-1}$ (2)

by virtue of the binomial expansion again,

$\omega = (2\pi / T_o)(1 + \frac{1}{2}\Gamma)$ (1)

Therefore, we complete the calculation:

$2\pi + \delta = \omega T_o$

$= (2\pi)(1 + \frac{1}{2}\Gamma)$

cancelling the 2π,

$\delta = \pi\Gamma$. (2)

Problem 4, continued

(c, 2)

$\delta = \pi \Gamma$

$= 6\pi v^2/c^2$

From $F = ma$ above, we may approximate v^2 with GM/R, (keeping the Γ from $F = ma$ would give δ here to second order in Γ), so that

$\delta = 6\pi GM / Rc^2$ (2)

(d, 4)

For one lap around the Sun, from the given numerical data,

$\delta_1 = 6\pi GM/Rc^2$

$= 6\pi (6.7\times10^{-11}\ \mathrm{Nm^2/kg^2})(2.0\times10^{30}\ \mathrm{kg})(3.0\times10^8\ \mathrm{m/s})^{-2}(5.8\times10^{10}\ \mathrm{m})^{-1}$

$= 4.8\times10^{-7}$ radians (2)

In 100 years, Mercury makes

$(365 / 88) \times 100 = 415$ laps around the Sun. (1)

For 415 laps,

$\delta_{415} = 415\ (4.8\times10^{-7}$ radians)

$= 2\times10^{-4}$ radians

$= (2\times10^{-4})(180/\pi)$ degrees

$= (2\times10^{-4})(180/\pi)\ (3600'')$

$= 41''$. (1)